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Coulomb drag in a longitudinal magnetic field in quantum wells

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Abstract

The influence of a longitudinal magnetic field on the Coulomb drag current created in the ballistic transport regime in a quantum well by a ballistic current in a nearby parallel quantum well is investigated. We consider the case where the magnetic field is so strong that the magnetic length $a_{\rm B}$ is smaller than the width of the well. Both in the ohmic and non-ohmic case, sharp peaks of the drag current as a function of the gate voltage or chemical potential are predicted. We study the dependence of the drag current on the voltage V across the driving wire, as well as on the magnetic field B. The fine structure of the peaks due to the electron spin is also considered. By studying the Coulomb drag one can make conclusions about the electron spectrum, g-factor and electron–electron interaction in quantum wells.

1. Introduction

The influence of a magnetic field on the Coulomb drag is investigated in different geometries. The Coulomb drag between two two-dimensional (2D) quantum wells in a strong magnetic field perpendicular to the planes of the wells and in the presence of disorder has been investigated in [1]. In a magnetic field perpendicular to the planes the Hall voltage can be induced in the drag quantum well in the direction perpendicular to both the direction of the magnetic field and of the current in the drive well [2, 3]. These two geometries can be called transverse.

The purpose of the present paper is to study the influence of an in-well magnetic field **B** on the Coulomb drag current in the course of ballistic (collisionless) electron transport in a quantum well due to a ballistic drive current in a parallel quantum well. In other words, we consider the longitudinal geometry, i.e. the case where the magnetic field is parallel to the applied electric field **E** and to the plane of the well itself.

We will concern ourselves with the case of a strong magnetic field that makes the motion of the carriers along the field one dimensional and alters the density of electron states. Moreover, we restrict ourselves with the quantum limit when only the ground Landau oscillator states are occupied by electrons in the two quantum wells, so that

$$\hbar\omega_{\rm B} \gtrsim \mu. \tag{1.1}$$

Here $\omega_{\rm B}$ is the cyclotron frequency while μ is the chemical potential. A theory of electronic transport through three-dimensional ballistic microwires in longitudinal magnetic fields at low temperatures has been developed in [4]. Our geometry is similar to that considered in [4]. However, in the present paper we consider the much simpler situation of a very strong magnetic field satisfying equation (1.1). Later on we hope to return to a more general case of a weaker magnetic field where several Landau levels may be involved.

The magnetic field making the motion of the electrons in the transverse direction one dimensional maps the problem under consideration onto the Coulomb drag problem in two one-dimensional wires already considered in [5, 6] in the Fermi liquid approach. Therefore, our final formulae for the Coulomb drag current appear to be similar to those obtained in [6]. Actually this approach is not always valid for the quantum wires. For the simplest case where in each wire only a single channel is active in the conductance the Luttinger liquid approach is often used. It should be applied provided the reservoirs do not destroy the Luttinger liquid. (In contrast, when several channels are active the Fermi liquid approach can be used.)

For the Coulomb drag in quantum wells the Fermi liquid approach is usually feasible. One may expect that it can also be applied to the wells under the influence of an in-plane magnetic field. Then we are left with a one-dimensional Fermi liquid problem that can be readily solved by perturbation theory. Its comparison with experiment is called for.

Physically the magnetic field may also play the following important role. It will suppress the tunnelling of electrons between the quantum wells that, if present, would impede observation of the Coulomb drag (see the analysis in section 6).

Note that the magnetic field may change the electron quasimomentum relaxation time. Scattering of electrons by ionized impurities in sufficiently strong magnetic fields may be even weaker than for $B = 0$ [7]. As for the relaxation due to the phonon scattering, a strong magnetic field can alter the density of electron states and in the quantum limit the relaxation rate may be bigger than for $B = 0$. We, however, will assume the temperature to be so low that the transport remains ballistic even in the presence of a magnetic field.

We consider the case where the magnetic length

$$a_{\rm B} = \sqrt{\frac{\hbar c}{|e|B}} \tag{1.2}$$

is much smaller than the distance W between quantum wells of width $L_x \sim W$ each:

$$W/a_{\rm B} \gg 1. \tag{1.3}$$

This inequality establishes a lower bound for the values of the magnetic field for a given distance between the quantum wells. For instance, for $W \sim 80$ nm the inequality requires magnetic fields of the order of $B \sim 1$ T, or bigger.

It is convenient to break our calculations into several parts. In the first part we will give the principal equations of our theory based on the Boltzmann treatment of the transport. We will consider a linear response in section 3. Next we will discuss a non-ohmic case in section 4. Comparison of our results with the 1D Coulomb drag results in the longitudinal geometry and 2D Coulomb drag results for $B = 0$ will be given in section 6.

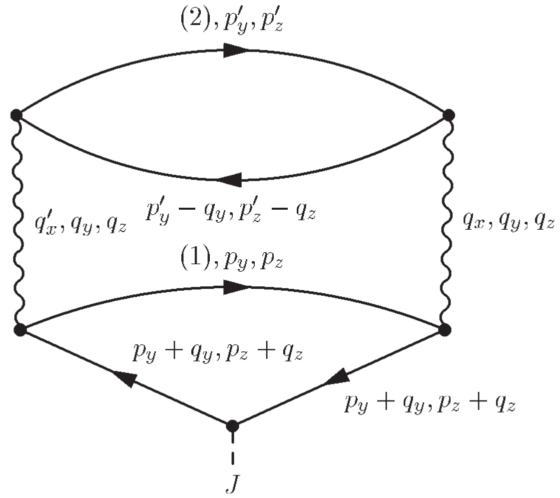

Figure 1. Coulomb drag diagram. Here the labels (2), (1) stand for the drive (drag) quantum wells.

2. Boltzmann equation

We consider two parallel quantum wells perpendicular to the x axis. The eigenfunctions and eigenvalues for a one-electron problem in a magnetic field along the z axis in the ith quantum well is (we use the gauge $\mathbf{A} = (0, Bx, 0)$)

$$\psi_{0p_yp_z} = \frac{1}{\sqrt{L_yL_z}}\varphi_0\left(\frac{x - x_{p_y}}{a_{\mathrm{B}}}\right)\exp(\mathrm{i}p_yy/\hbar + \mathrm{i}p_zz/\hbar), \tag{2.1}$$

$$\varepsilon_{0p_z} = U_i + \frac{\hbar\omega_{\mathrm{B}}}{2} + \frac{p_z^2}{2m}. \tag{2.2}$$

Here m is the effective electron mass, and $\varphi_0[(x - x_{p_y})/a_{\mathrm{B}}]$ is the wavefunction of a harmonic oscillator in the ground state oscillating about the point $x_{p_y} = -a_{\mathrm{B}}^2p_y/\hbar = -p_y/m\omega_{\mathrm{B}}$. The wavefunction $\psi_{0p_yp_z}$ describes a state for which the electron probability distribution is large only within the slab of width $\approx a_{\mathrm{B}}$ symmetrically situated about the plane $x = x_{p_y}$, and falls off exponentially outside the slab. As we consider the case $L_x \gg a_{\mathrm{B}}$ we will assume the wavefunction to be equal to $\psi_{0p_yp_z}$ if x_{p_y} is within the quantum well and zero otherwise. In what follows we will need the matrix elements of the functions $\exp(\pm\mathrm{i}\mathbf{qr})$ between any two ground states of a Landau oscillator. We have

$$\langle p'_yp'_z|\mathrm{e}^{\pm\mathrm{i}\mathbf{qr}}|p_yp_z\rangle = \exp[\pm\mathrm{i}q_x(x_{p_y} + x_{p'_y})/2] \times \exp[-a_{\mathrm{B}}^2q_x^2/4]\exp[-(x_{p_y} - x_{p'_y})^2/4a_{\mathrm{B}}^2]\delta_{p'_z,p_z\pm\hbar q_z}\delta_{p'_y,p_y\pm\hbar q_y}. \tag{2.3}$$

A diagram representing the Coulomb drag effect is given in figure 1.

The external driving force enters the diagram through the nonequilibrium distribution function represented by the solid curves marked by the symbol (2) indicating that they represent the drive quantum well. Now we embark on an analysis of the conservation laws for the collisions of electrons belonging to two different quantum wells. We have

$$\varepsilon_{0p_z}^{(1)} + \varepsilon_{0p'_z}^{(2)} = \varepsilon_{0p_z+\hbar q_z}^{(1)} + \varepsilon_{0p'_z-\hbar q_z}^{(2)}, \tag{2.4}$$

where $\varepsilon_{0p_z}^{(1,2)} = U_{1,2} + \hbar\omega_{\mathrm{B}}/2 + p_z^2/2m$. Here $U_{1,2}$ describe the positions of the bottoms of the electron bands in the wires.

The solution of equation (2.4) is

$$\hbar q_z = p'_z - p_z. \tag{2.5}$$

The δ-function describing energy conservation can be recast into the form

$$\delta(\varepsilon^{(1)}_{0p_z} + \varepsilon^{(2)}_{0p'_z} - \varepsilon^{(1)}_{0p_z+\hbar q_z} - \varepsilon^{(2)}_{0p'_z-\hbar q_z}) = \frac{m}{\hbar|q_z|}\delta[\hbar q_z - (p'_z - p_z)]. \tag{2.6}$$

Therefore, the initial quasimomenta p_z and p'_z after the collision become $p_z + \hbar q_z = p'_z$ and $p'_z - \hbar q_z = p_z$, i.e. the electrons swap their quasimomenta as a result of the collision.

Following [5, 6] we assume that the drag current in quantum well 1 is much smaller than the drive ballistic current in quantum well 2 and calculate it by solving the Boltzmann equation for quantum well 1. We have

$$v_z \frac{\partial \Delta F^{(1)}_{0p_y}(p_z, z)}{\partial z} = -I^{(12)}\{F^{(1)}, F^{(2)}\}, \tag{2.7}$$

where $F^{(1,2)}$ are the electron distribution functions in quantum wells 1 and 2 respectively, and the collision integral $I^{(12)}\{F^{(1)}, F^{(2)}\}$ takes into account the interwell electron–electron scattering:

$$I^{(12)}\{F^{(1)}, F^{(2)}\} = \sum_{p'_z p'_y q'_x \mathbf{q}} W^{1p_z+\hbar q_z, 2p'_z-\hbar q_z}_{1p_z, 2p'_z}(q'_x, q_x, q_y, p_y, p'_y)\mathcal{S}. \tag{2.8}$$

In this expression the sum over p'_y should be determined by the requirement that the x-centre of the oscillator function is within the second quantum well. The requirement imposes the constraint

$$\frac{\hbar}{a_{\mathrm{B}}^2}\left(W + L_x - \frac{L_x}{2}\right) < p'_y < \frac{\hbar}{a_{\mathrm{B}}^2}\left(W + L_x + \frac{L_x}{2}\right), \tag{2.9}$$

and the product of distribution functions $\mathcal{S}$ is

$$\mathcal{S} = F^{(1)}_{0p_z}F^{(2)}_{0p'_z}(1 - F^{(1)}_{0p_z+\hbar q_z})(1 - F^{(2)}_{0p'_z-\hbar q_z}) - F^{(1)}_{0p_z+\hbar q_z}F^{(2)}_{0p'_z-\hbar q_z}(1 - F^{(1)}_{0p_z})(1 - F^{(2)}_{0p'_z}) \tag{2.10}$$

$$F^{(1)}_{0p_z} = \theta[v_z]f(\varepsilon_{0p_z} - \mu^{1\mathrm{L}}_{\mathrm{B}}) + \theta[-v_z]f(\varepsilon_{0p_z} - \mu^{1\mathrm{R}}_{\mathrm{B}}) + \Delta F^{(1)}_{0p_z} \tag{2.11}$$

where

$$\theta[v_z] = \begin{cases} 1 & \text{for } v_z > 0 \\ 0 & \text{for } v_z < 0. \end{cases}$$

Here we assume that the electrons move ballistically within the quantum well (except, of course, the interwell Coulomb scattering). The electrons moving from the left and right reservoirs have the chemical potentials $\mu^{1\mathrm{L}}_{\mathrm{B}} = \mu_{\mathrm{B}} - eV_{\mathrm{d}}/2$ and $\mu^{1\mathrm{R}}_{\mathrm{B}} = \mu_{\mathrm{B}} + eV_{\mathrm{d}}/2$ respectively. We also introduce the drag voltage V_{d} induced across the drag quantum well due to the quasimomentum transfer from the driving quantum well, i.e. we assume an open circuit for the drag quantum well.

The solution of equation (2.7) is (here we omit the equilibrium part)

$$\Delta F^{(1)}_{0p_z} = -\left(z \pm \frac{L_z}{2}\right)\frac{1}{v_z}I^{(12)}\{F^{(1)}, F^{(2)}\}, \qquad \text{for } \begin{array}{l} p_z > 0, \\ p_z < 0. \end{array} \tag{2.12}$$

Using the particle conserving property of the scattering integral

$$\sum_{p_z p_y} I^{(12)}\{F^{(1)}, F^{(2)}\} = 0 \tag{2.13}$$

we get for the total current in the drag quantum well defined as

$$J = \frac{e}{L_z} \sum_{p_y p_z} v_z F^{(1)}_{0p_z}, \tag{2.14}$$

the result

$$J = -e \sum_{p_y,(p_z>0)} I^{(12)}\{F^{(1)}, F^{(2)}\} + e\frac{1}{L_z} \sum_{p_y,(p_z>0)} v_z[f(\varepsilon_{0p_z} - \mu_B^{1L}) - f(\varepsilon_{0p_z} - \mu_B^{1R})]. \tag{2.15}$$

In these equations the sum over p_y is restricted by the requirement that the x-centre of the Landau oscillator must be within the quantum well, so that $-\hbar L_x/2a_B^2 < p_y < \hbar L_x/2a_B^2$. Introducing the density of states (including spin) per unit quasimomentum interval

$$N(p_z)\,\mathrm{d}p_z = 2\frac{L_z}{(2\pi\hbar)^2}\frac{\hbar L_x L_y}{a_B^2}\,\mathrm{d}p_z \tag{2.16}$$

we have

$$J_{\mathrm{Ohm}} = -e\frac{2eV_\mathrm{d}}{(2\pi\hbar)^2}\frac{\hbar L_x L_y}{a_B^2}\int_{U_1+\hbar\omega_B/2}^{\infty} \mathrm{d}\varepsilon \left(-\frac{\partial f(\varepsilon-\mu_B)}{\partial\varepsilon}\right). \tag{2.17}$$

For the degenerate electron gas this expression can be written as

$$J_{\mathrm{Ohm}} = -\frac{e^2}{\pi\hbar}\frac{L_x L_y}{2\pi a_B^2} V_\mathrm{d}. \tag{2.18}$$

Here the number of Larmor circles covering the cross section of the quantum well $L_x L_y/2\pi a_B^2$ appears instead of the number of open channels in the 1D situation. (We will sometimes use the term 'Larmor circles', more appropriate for the classical limit, also for the quantum limit considered here.)

We assume that only the ground Landau oscillator state is occupied, so that

$$U_1 + \tfrac{1}{2}\hbar\omega_B < \mu_B < U_1 + \tfrac{3}{2}\hbar\omega_B. \tag{2.19}$$

Taking into account equation (2.6) we obtain for the Coulomb scattering probability equation (2.8)

$$\begin{aligned}
W^{1p_z+\hbar q_z,2p'_z-\hbar q_z}_{1p_z,2p'_z}(q'_x, q_x, q_y, p_y, p'_y) &= \frac{m}{\hbar|q_z|}\delta[\hbar q_z - (p'_z - p_z)]\frac{2\pi}{\hbar}U_\mathbf{q}U_{q'_x q_y q_z} \\
&\times \langle p_z, p_y|\mathrm{e}^{-\mathrm{i}q'_x x - \mathrm{i}q_y y - \mathrm{i}q_z z}|p_z + \hbar q_z, p_y + \hbar q_y\rangle \\
&\times \langle p_z + \hbar q_z, p_y + \hbar q_y|\mathrm{e}^{\mathrm{i}q_x x + \mathrm{i}q_y y + \mathrm{i}q_z z}|p_z, p_y\rangle \\
&\times \langle p'_z, p'_y|\mathrm{e}^{\mathrm{i}q'_x x + \mathrm{i}q_y y + \mathrm{i}q_z z}|p'_z - \hbar q_z, p'_y - \hbar q_y\rangle \\
&\times \langle p'_z - \hbar q_z, p'_y - \hbar q_y|\mathrm{e}^{-\mathrm{i}q_x x - \mathrm{i}q_y y - \mathrm{i}q_z z}|p'_z, p'_y\rangle.
\end{aligned} \tag{2.20}$$

Here we use the unscreened Coulomb potential, postponing discussion as to when this approximation can be justified until the last section.

To calculate the drag current we iterate the Boltzmann equation in the interwell collision term that we assume to be small. Therefore one can choose the distribution functions in the collision term to be equilibrium ones, e.g. $F^{(1)}_{0p} = f(\varepsilon^{(1)}_{0p} - \mu_B)$ for the first quantum well.

We assume, in the spirit of the approach developed by Landauer [8], Imry [9] and Büttiker [10], the drive quantum well to be connected to reservoirs which we call 'left' l and 'right' r. Each of them is in independent equilibrium described by the shifted chemical potentials $\mu_B^\mathrm{l} = \mu_B - eV/2$ and $\mu_B^\mathrm{r} = \mu_B + eV/2$, where μ_B is the equilibrium chemical potential in the magnetic field. Therefore, the electrons entering the quantum well from the 'left' ('right') and having quasimomenta $p'_z > 0\,(p'_z < 0)$ are described by $F^{(2)}_{0p'_z} = f(\varepsilon^{(2)}_{0p'_z} - \mu_B^\mathrm{l})$

$[F^{(2)}_{0p'_z} = f(\varepsilon^{(2)}_{0p'_z} - \mu^{\rm r}_{\rm B})]$ and we see that the collision integral equation (2.8) is identically zero if the initial p'_z and final $p'_z - \hbar q_z$ quasimomenta in the drive quantum well are of the same sign. This means that only the *backscattering processes* contribute to the drag current.

Due to equation (2.6) we are left only with $p'_z < 0$ (since we are restricted according to equation (2.15) by the constraint $p'_z - \hbar q_z = p_z > 0$) and obtain in view of the δ-function in equation (2.20) the following product of distribution functions in the collision term:

$$\mathcal{P} = F^{(1)}_{0p_z} F^{(2)r}_{0p'_z}(1 - F^{(1)}_{0p'_z})(1 - F^{(2)l}_{0p_z}) - F^{(1)}_{0p'_z} F^{(2)l}_{0p_z}(1 - F^{(1)}_{0p_z})(1 - F^{(2)r}_{0p'_z}), \qquad (2.21)$$

or

$$\mathcal{P} = f(\varepsilon^{(1)}_{0p_z} - \mu_{\rm B}) f(\varepsilon^{(2)}_{0p'_z} - \mu^{\rm r}_{\rm B})[1 - f(\varepsilon^{(1)}_{0p'_z} - \mu_{\rm B})][1 - f(\varepsilon^{(2)}_{0p_z} - \mu^{\rm l}_{\rm B})]$$
$$- f(\varepsilon^{(1)}_{0p'_z} - \mu_{\rm B}) f(\varepsilon^{(2)}_{0p_z} - \mu^{\rm l}_{\rm B})[1 - f(\varepsilon^{(1)}_{0p_z} - \mu_{\rm B})][1 - f(\varepsilon^{(2)}_{0p'_z} - \mu^{\rm r}_{\rm B})]. \qquad (2.22)$$

This equation will be analysed in the following sections.

3. Linear response

In this case $eV/T \ll 1$ (we assume the Boltzmann constant to be equal to 1) and equation (2.22) can be recast into the form

$$\mathcal{P} = \frac{eV}{T} f(\varepsilon^{(1)}_{0p_z} - \mu_{\rm B}) f(\varepsilon^{(2)}_{0p'_z} - \mu_{\rm B})[1 - f(\varepsilon^{(1)}_{0p'_z} - \mu_{\rm B})][1 - f(\varepsilon^{(2)}_{0p_z} - \mu_{\rm B})]. \qquad (3.1)$$

Shifting the integration variable $p'_y \to p'_y + \hbar(W + L_x)/a^2_{\rm B}$ we have for the drag current

$$J_{\rm drag} = -e\frac{eV}{T}\frac{2\pi}{\hbar}\left(\frac{4\pi e^2}{\kappa}\right)^2 \frac{1}{2\pi\hbar}\int_0^\infty \frac{2L_z\,{\rm d}p_z}{2\pi\hbar}\int_0^\infty \frac{{\rm d}p'_z}{2\pi\hbar}\frac{m}{(p_z + p'_z)}$$
$$\times f(\varepsilon^{(1)}_{0p_z} - \mu_{\rm B}) f(\varepsilon^{(2)}_{0p'_z} - \mu_{\rm B})[1 - f(\varepsilon^{(1)}_{0p'_z} - \mu_{\rm B})][1 - f(\varepsilon^{(2)}_{0p_z} - \mu_{\rm B})]$$
$$\times \int_{-\hbar L_x/2a^2_{\rm B}}^{\hbar L_x/2a^2_{\rm B}} \frac{2L_y\,{\rm d}p_y}{2\pi\hbar}\frac{{\rm d}p'_y}{2\pi\hbar} g_{00}\left[\frac{p_z + p'_z}{\hbar}, \frac{p_y - p'_y}{\hbar}\right] \qquad (3.2)$$

where κ is the dielectric susceptibility and

$$g_{00}(k_z, k_y) = \int_{-\infty}^\infty \frac{{\rm d}q_y}{2\pi}{\rm e}^{-a^2_{\rm B}q^2_y} A^2(k_z, k_y, q_y), \qquad (3.3)$$

$$A(k_z, k_y, q_y) = \int_{-\infty}^\infty \frac{{\rm d}q_x}{2\pi}\frac{{\rm e}^{-{\rm i}a^2_{\rm B}q_x[k_y - (W+L_x)/a^2_{\rm B} + q_y]}{\rm e}^{-a^2_{\rm B}q^2_x/2}}{(k^2_z + q^2_\perp)}, \qquad (3.4)$$

$q^2_\perp = q^2_x + q^2_y$. The last integral over the centres of the Landau oscillators (since $x_{p_y} = -a^2_{\rm B}p_y/\hbar$) in equation (3.2) plays the role of an effective Coulomb interaction potential between the electrons freely moving along the direction of applied magnetic field.

$$\int_{-\hbar L_x/2a^2_{\rm B}}^{\hbar L_x/2a^2_{\rm B}} \frac{2L_y\,{\rm d}p_y}{2\pi\hbar}\frac{{\rm d}p'_y}{2\pi\hbar} g_{00}\left[\frac{p_z + p'_z}{\hbar}, \frac{p_y - p'_y}{\hbar}\right]$$
$$= \frac{2L_y}{(2\pi)^2}\int_0^{L_x/a^2_{\rm B}} {\rm d}k_y k_y\left\{g_{00}\left[\frac{p_z + p'_z}{\hbar}, \frac{L_x}{a^2_{\rm B}} - k_y\right] + g_{00}\left[\frac{p_z + p'_z}{\hbar}, k_y - \frac{L_x}{a^2_{\rm B}}\right]\right\}. \qquad (3.5)$$

We keep only the first term in this expression since the second term includes a faster oscillating exponent $\sim\exp[{\rm i}q_x(W + L_x)]$ as compared to the oscillating exponent in the first term $\sim\exp({\rm i}q_x W)$.

As $W/a_{\mathrm{B}} \gg 1$, we can sufficiently simplify the expression for g_{00}. We obtain

$$g_{00}(k_z, k_y) = \exp(a_{\mathrm{B}}^2 k_z^2) \int_{-\infty}^{\infty} \frac{\mathrm{d}q_y}{2\pi} A^2(k_z, k_y, q_y). \tag{3.6}$$

$$A(k_z, k_y, q_y) \simeq \int \frac{\mathrm{d}q_x}{2\pi} \frac{\mathrm{e}^{\mathrm{i}q_x[(W+L_x)-a_{\mathrm{B}}^2 k_y]}}{q_x^2 + q_y^2 + k_z^2} = \frac{\mathrm{e}^{-|W+L_x-a_{\mathrm{B}}^2 k_y|\sqrt{q_y^2+k_z^2}}}{2\sqrt{q_y^2 + k_z^2}}. \tag{3.7}$$

Finally, the interaction term acquires the form

$$\int_{-\hbar L_x/2a_{\mathrm{B}}^2}^{\hbar L_x/2a_{\mathrm{B}}^2} \frac{2L_y\,\mathrm{d}p_y}{2\pi\hbar} \frac{\mathrm{d}p'_y}{2\pi\hbar} g_{00}\left[k_z, \frac{p_y - p'_y}{\hbar}\right] = \frac{L_y}{4a_{\mathrm{B}}(2\pi a_{\mathrm{B}} k_z)^3} \exp(a_{\mathrm{B}}^2 k_z^2)\Phi(2Wk_z), \tag{3.8}$$

where

$$\Phi(\alpha) = \int_1^{\infty} \mathrm{d}\xi \frac{\mathrm{e}^{-\alpha\xi}}{\xi^3\sqrt{\xi^2 - 1}}. \tag{3.9}$$

For $\alpha \gg 1$,

$$\Phi(\alpha) \simeq \sqrt{\frac{\pi}{2\alpha}}\mathrm{e}^{-\alpha}. \tag{3.10}$$

This result for the effective interaction (3.8) can be explained as follows: the Larmor circles within the layers of width $L_x \cdot 1/(k_z L_x)$ near the surfaces contribute to the interaction. The number of interacting circles from two quantum wells is

$$\left(\frac{L_y}{a_{\mathrm{B}}} \frac{L_x \cdot 1/(k_z L_x)}{a_{\mathrm{B}}}\right)^2.$$

The sum over $q_x, q'_x \sim k_z, q_y \sim \sqrt{k_z/W}$ brings the factor $(k_z L_x)^2 \cdot L_y \sqrt{k_z/W}$. The exponential decay of the drag with the distance between the quantum wells W is a consequence of the one-dimensional character of the drag in the strong longitudinal magnetic field. Combining all these factors and multiplying the result by $U^2 \sim (4\pi e^2)^2/(L_x L_y L_z)^2 k_z^4$ we arrive at equation (3.8).

The product of the distribution functions in equation (3.2) is a sharp function of p_z and p'_z at small temperatures, acquiring nonzero values provided that p_z, p'_z are equal to $p_{\mathrm{F}}^{\mathrm{B}}$ with the accuracy $T/v_{\mathrm{F}}^{\mathrm{B}}$. We assume that the quasimomentum interval $T/v_{\mathrm{F}}^{\mathrm{B}}$ is much smaller than $\hbar/W$:

$$T \ll \frac{\hbar v_{\mathrm{F}}^{\mathrm{B}}}{W}. \tag{3.11}$$

Here we wish to note that the Boltzmann treatment of transport phenomena requires that the uncertainty in the longitudinal quasimomentum $\hbar/L_z$ must be smaller than the quasimomentum interval $T/v_{\mathrm{F}}^{\mathrm{B}}$, i.e. $\hbar/L_z \ll T/v_{\mathrm{F}}^{\mathrm{B}}$. These two requirements automatically lead to the inequality $W \ll L_z$. We assume that the last inequality holds.

According to our assumptions we can regard the interaction term in equation (3.2) as slowly varying and obtain

$$J_{\mathrm{drag}} = J_0 \frac{eV}{4\varepsilon_{\mathrm{F}}^{\mathrm{B}}} \frac{T}{\varepsilon_{\mathrm{F}}^{\mathrm{B}}} \left(\frac{U_{12}}{2T}\right)^2 \left[\sinh\left(\frac{U_{12}}{2T}\right)\right]^{-2}, \tag{3.12}$$

where

$$J_0 = -\frac{e^5 m}{\kappa^2(4\pi\hbar)^3} \frac{L_z L_y}{a_{\mathrm{B}}^2} \frac{1}{(a_{\mathrm{B}} k_{\mathrm{F}}^{\mathrm{B}})^2} \mathrm{e}^{(2a_{\mathrm{B}} k_{\mathrm{F}}^{\mathrm{B}})^2} \Phi\left(4Wk_{\mathrm{F}}^{\mathrm{B}}\right). \tag{3.13}$$

Here we introduced notations $U_{12} = U_1 - U_2$ and $m v_{\mathrm{F}}^{\mathrm{B}} = p_{\mathrm{F}}^{\mathrm{B}} = \sqrt{2m[\mu_{\mathrm{B}} - U_1 - \hbar\omega_{\mathrm{B}}/2]}$, $k_{\mathrm{F}}^{\mathrm{B}} = p_{\mathrm{F}}^{\mathrm{B}}/\hbar$.

We assume that the electrons remain degenerate in the magnetic field:

$$\varepsilon_{\mathrm{F}}^{\mathrm{B}} \equiv \mu_{\mathrm{B}} - U_1 - \frac{\hbar\omega_{\mathrm{B}}}{2} \gg T. \tag{3.14}$$

The quantum limit is considered, i.e. the case where all the electrons belong to the first Landau level:

$$\varepsilon_{\mathrm{F}}^{\mathrm{B}} < \hbar\omega_{\mathrm{B}}. \tag{3.15}$$

Since the electron concentration N_{B} under this condition is related to the chemical potential by the equation

$$N_{\mathrm{B}} = \frac{m\hbar\omega_{\mathrm{B}} p_{\mathrm{F}}^{\mathrm{B}}}{\pi^2\hbar^3} \tag{3.16}$$

equations (3.14) and (3.15) lead to

$$T \ll \frac{(p_{\mathrm{F}}^{\mathrm{B}})^2}{2m} < \hbar\omega_{\mathrm{B}}, \qquad p_{\mathrm{F}}^{\mathrm{B}} = \frac{\pi^2\hbar^3}{m}\frac{N_{\mathrm{B}}}{\hbar\omega_{\mathrm{B}}}. \tag{3.17}$$

The first inequality in this relation is weaker than equation (3.11) if $\varepsilon_{\mathrm{F}} \sim \hbar\omega_{\mathrm{B}}$ and $Wk_{\mathrm{F}} \gtrsim 1$. Introducing the electron concentration N and the chemical potential μ for $B = 0$ given by

$$N = \frac{(2m\varepsilon_{\mathrm{F}})^{3/2}}{3\pi^2\hbar^3}, \qquad \varepsilon_{\mathrm{F}} = \mu - U_1 \tag{3.18}$$

one can rewrite equation (3.17) as

$$T \ll \frac{4}{9}\left(\frac{N_{\mathrm{B}}}{N}\right)^2 \left(\frac{\varepsilon_{\mathrm{F}}}{\hbar\omega_{\mathrm{B}}}\right)^2 \varepsilon_{\mathrm{F}} < \hbar\omega_{\mathrm{B}}. \tag{3.19}$$

Note that the second inequality in this expression does not depend on the electron mass and can require magnetic fields stronger than equation (1.3) (thus imposing a constraint on the electron concentration, or, if the latter is given the inequality may require stronger magnetic fields than is required by equation (1.3)). For instance, in a magnetic field of the order of $B \sim 10$ T the electron concentration N must be smaller than 2.7×10^{17} cm^{-3}.

Considering the case of the aligned quantum wells, so that $U_1 = U_2$ (otherwise the effect is exponentially small; see equation (3.12)) and putting $N = N_{\mathrm{B}}$ we obtain

$$J_{\mathrm{drag}} = J_0 \frac{eV}{4T}\left(\frac{T}{\varepsilon_{\mathrm{F}}}\right)^2 \left(\frac{3\hbar\omega_{\mathrm{B}}}{2\varepsilon_{\mathrm{F}}}\right)^4, \tag{3.20}$$

$$J_0 = -\frac{e^5 m L_y L_z k_{\mathrm{F}}^2}{9\kappa^2(4\pi\hbar)^3}\left(\frac{3\hbar\omega_{\mathrm{B}}}{2\varepsilon_{\mathrm{F}}}\right)^4 \exp[12(2\varepsilon_{\mathrm{F}}/3\hbar\omega_{\mathrm{B}})^3]\Phi\left(4Wk_{\mathrm{F}}\frac{2\varepsilon_{\mathrm{F}}}{3\hbar\omega_{\mathrm{B}}}\right). \tag{3.21}$$

The drag current is a rapidly increasing function of the applied magnetic field, as the latter increases the density of states and decreases the transferred Fermi momentum (see figure 2).

To make an estimate of the current we put $m = 0.07m_0$ (where m_0 is the free electron mass), $\hbar\omega_{\mathrm{B}} \sim \varepsilon_{\mathrm{F}} = 14$ meV, $\kappa = 13$, $L_z \sim L_y = 1$ μm, and $W = 40$ nm.

$$J_{\mathrm{drag}} \sim 10^{-11} \text{ A}.$$

In the linear response regime we can introduce a drag resistance, i.e. the coefficient that depends only on the quantum well parameters and relates the drive current J_{drive} in quantum well 2 to the induced voltage in the drag quantum well $J_{\mathrm{drive}} R_{\mathrm{D}} = V_{\mathrm{d}}$. Here the drive current in quantum well 2 is

$$J_{\mathrm{drive}} = -V\frac{e^2}{2\pi\hbar}\frac{L_x L_y}{\pi a_{\mathrm{B}}^2} \tag{3.22}$$

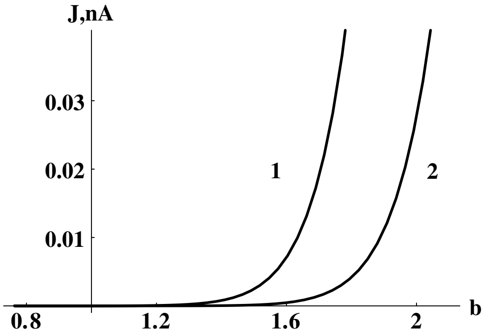

Figure 2. Drag current versus dimensionless magnetic field $b = \hbar\omega_B/\varepsilon_F$ for two values of the interwell distances $W = 40$ nm (1) and $W = 50$ nm (2). The other parameters are given in the text.

(see equation (2.18)), and V_d is determined by the condition of zero total current $J = J_{drag} + J_{Ohm} = 0$ in the drag quantum well in equation (2.15).

$$R_D = \frac{\pi\hbar}{e^2}\frac{E_B}{\varepsilon_F}\frac{T}{\varepsilon_F}\frac{L_z}{L_y}\frac{1}{(k_F L_x)^2}\left(\frac{3\hbar\omega_B}{4\varepsilon_F}\right)^6 \exp[12(2\varepsilon_F/3\hbar\omega_B)^3]\Phi\left[4Wk_F\frac{2}{3}\frac{\varepsilon_F}{\hbar\omega_B}\right], \tag{3.23}$$

where we introduced the effective Bohr energy $E_B = me^4/\kappa^2\hbar^2$ and

$$p_F^B = \frac{2}{3}\frac{N_B}{N}\frac{\varepsilon_F}{\hbar\omega_B}p_F, \qquad p_F = \sqrt{2m\varepsilon_F}. \tag{3.24}$$

With the parameters given above we have the following estimate for the transresistance:

$$R_D \sim 0.4 \text{ m}\Omega.$$

Now let us discuss when one can neglect the screening. Since the transferred momenta are $q_z \sim 2p_F^B/\hbar$ it is permissible not to take into account the screening of the Coulomb potential if the inverse screening length is much smaller than the transferred momentum. The dielectric function in our case has contributions from both the intrawell and the interwell Coulomb interactions. Neglecting the small interwell contribution it can be written as

$$\epsilon(\omega, q_z) = |1 - U_{\mathbf{q}}\Pi^R(\omega, \mathbf{q})|^2, \tag{3.25}$$

where Π^R is the retarded polarization function. We can estimate the latter at transferred energies $\sim T$ and momenta $\sim 2p_F^B$ regarding the electron motion as one dimensional:

$$\Pi^R(T, 2p_F^B) \simeq \frac{L_x L_y}{2\pi a_B^2}\frac{mL_z}{\pi\hbar p_F^B}\ln\frac{\varepsilon_F^B}{T}. \tag{3.26}$$

Therefore one can estimate the screening length from

$$\frac{1}{r_s^2} \sim \frac{4\pi e^2}{\kappa L_x L_y L_z}\Pi^R(T, 2\pi_F^B) \tag{3.27}$$

as

$$\frac{1}{r_s} \sim \sqrt{\frac{\pi e^2 N_B}{\kappa\varepsilon_F^B}\ln\frac{\varepsilon_F^B}{T}}. \tag{3.28}$$

The required inequality can be written as (we put $N_B = N$)

$$\frac{N^{1/3}e^2}{\kappa}\ln\left[\frac{\varepsilon_F}{T}\left(\frac{\varepsilon_F}{\hbar\omega_B}\right)^2\right] \ll \varepsilon_F\left(\frac{\varepsilon_F}{\hbar\omega_B}\right)^4. \tag{3.29}$$

We will assume this inequality to be satisfied.

4. Non-ohmic case

The product of distribution functions in equation (2.22) can be recast into the form

$$\mathcal{P} = 2\sinh(eV/2T)\exp\{(\varepsilon^{(1)}_{p_z} - \mu_B)/T\}\exp\{(\varepsilon^{(2)}_{p_z'} - \mu_B)/T\} f(\varepsilon^{(1)}_{p_z} - \mu_B) \times f(\varepsilon^{(2)}_{p'_z} - \mu_B - eV/2) f(\varepsilon^{(1)}_{p'_z} - \mu_B) f(\varepsilon^{(2)}_{p_z} - \mu_B + eV/2). \quad (4.1)$$

As $\mathcal{P}$ is a sharp function of p_z and p'_z one can take out of the integral all the slowly varying functions and get

$$\int_0^\infty \mathrm{d}p_z\,\mathrm{d}p'_z \frac{\mathcal{P}}{(p_z + p'_z)} \int_{-\hbar L_x/2a_B^2}^{\hbar L_x/2a_B^2} \frac{2L_y\,\mathrm{d}p_y}{2\pi\hbar}\frac{\mathrm{d}p'_y}{2\pi\hbar} g_{00}\left[(p_z + p'_z)/\hbar, (p_y - p'_y)/\hbar\right] = \frac{L_y m^2 a_B^2 T^2 \exp[(2a_B k_F^B)^2]}{4(4\pi\hbar)^3 (a_B k_F^B)^6}\Phi(4Wk_F^B) \times \sinh\left(\frac{eV}{2T}\right)\frac{\frac{eV}{4T} - \frac{U_{12}}{2T}}{\sinh\left(\frac{eV}{4T} - \frac{U_{12}}{2T}\right)}\frac{\frac{eV}{4T} + \frac{U_{12}}{2T}}{\sinh\left(\frac{eV}{4T} + \frac{U_{12}}{2T}\right)}. \quad (4.2)$$

The drag current is

$$J_{\mathrm{drag}} = J_0\frac{1}{2}\left(\frac{T}{\varepsilon_F^B}\right)^2 \sinh\left(\frac{eV}{2T}\right)\frac{\frac{eV}{4T} - \frac{U_{12}}{2T}}{\sinh\left(\frac{eV}{4T} - \frac{U_{12}}{2T}\right)}\frac{\frac{eV}{4T} + \frac{U_{12}}{2T}}{\sinh\left(\frac{eV}{4T} + \frac{U_{12}}{2T}\right)}. \quad (4.3)$$

For $eV \ll T$ one gets equation (3.20). In the opposite case $eV \gg T$ one gets a nonvanishing result for equation (4.3) only provided $|U_{12}| < eV/2$. One obtains the following equation for the drag current:

$$J_{\mathrm{drag}} = J_0\left[\left(\frac{eV}{4\varepsilon_F}\right)^2 - \left(\frac{U_{12}}{2\varepsilon_F}\right)^2\right]\left(\frac{3\hbar\omega_B}{2\varepsilon_F}\right)^4. \quad (4.4)$$

Thus the drag current vanishes unless $eV > 2|U_{12}|$.

5. Spin effects

Let us discuss how the Zeeman splitting by an in-plane magnetic field

$$H_Z = g\mu_B \mathbf{Bs}, \quad (5.1)$$

where μ_B is the Bohr magneton, will modify our results. The in-plane g-factor for the quantum wells of GaAs/AlGaAs heterostructures in the wide well limit is close to that of the bulk GaAs, where $g = -0.44$. For narrower quantum wells according to Ivchenko *et al* [11] its absolute value is smaller and the g-factor can even change the sign.

For electron energies including this splitting we have

$$\varepsilon^{(1,2)}_{0p_z\pm} = U_{1,2} + \frac{p_z^2}{2m} + \frac{\hbar\omega_B}{2}(1 \pm \Delta), \quad (5.2)$$

where the dimensionless energy shift $\Delta = gm/2m_0$ includes the ratio of the effective electron mass m to the free electron mass m_0. As in GaAs the ratio is small, $m \approx 0.07m_0$, we conclude that the shift $|\Delta| \ll 1$. However, the splitting proportional to $\hbar\omega_B\Delta$ may be important since this energy shift enters our equations along with the energy band bottom difference U_{12} that we also consider to be small. Therefore, instead of equation (3.12) we arrive at a similar formula having a sort of 'fine' structure:

$$J_{\rm drag} = J_0 \frac{eV}{16\varepsilon_{\rm F}^{\rm B}} \frac{T}{\varepsilon_{\rm F}^{\rm B}} \left\{ 2\left(\frac{U_{12}}{2T}\right)^2 \left[\sinh\left(\frac{U_{12}}{2T}\right)\right]^{-2} + \left(\frac{U_{12} + \hbar\omega_{\rm B}\Delta}{2T}\right)^2 \left[\sinh\left(\frac{U_{12} + \hbar\omega_{\rm B}\Delta}{2T}\right)\right]^{-2} + \left(\frac{U_{12} - \hbar\omega_{\rm B}\Delta}{2T}\right)^2 \left[\sinh\left(\frac{U_{12} - \hbar\omega_{\rm B}\Delta}{2T}\right)\right]^{-2} \right\}. \quad (5.3)$$

Investigation of this 'fine' structure provides one more method of measurement of the in-plane g-factor.

6. Concluding remarks

It is interesting to compare our results with two different experimental geometries. First, let us consider the influence of magnetic field on 1D Coulomb drag for the longitudinal geometry. In this case the magnetic field is directed along the z axis and is parallel to 1D nanowires. For simplicity, we assume that the confining potential in the absence of the magnetic field is

$$U(x, y) = \frac{m\Omega^2}{2}(x^2 + y^2), \quad (6.1)$$

where Ω is the eigenfrequency of electron oscillation in the potential $U(x, y)$. The applied magnetic field shortens the radius of the state so that it becomes

$$a_{\rm B}^2 = \frac{a_0^2}{\sqrt{1 + (B/B_{\rm c})^2}}, \qquad B_{\rm c} = 2\frac{\Omega mc}{|e|}, \qquad a_0 = \sqrt{\frac{\hbar}{2m\Omega}}, \quad (6.2)$$

where a_0 is the radius in the absence of the magnetic field. For the lowest Landau level we have

$$\phi = \frac{1}{\sqrt{2\pi}} \frac{1}{a_{\rm B}} \exp(-\rho^2/4a_{\rm B}^2), \qquad \varepsilon_{\rm p} = \frac{\hbar^2}{2ma_{\rm B}^2} + \frac{p_z^2}{2m}. \quad (6.3)$$

The wavefunction of the electron in the second wire can be obtained by a gauge transformation of the wavefunction in the first one. Since the interaction term g_{00} is not phase sensitive we are left only with a shift by the distance W between the centres of the wires in the argument of the wavefunction (6.3). As a result, one gets for the interaction

$$g_{00}(2p_{\rm F}^{\rm B}) = 4{\rm e}^{-W^2/2a_{\rm B}^2} \left[\int_0^\infty {\rm d}\rho\, \rho {\rm e}^{-\rho^2} I_0\left(\frac{W}{a_{\rm B}}\rho\right) K_0\left(4\frac{p_{\rm F}^{\rm B} a_{\rm B}}{\hbar}\rho\right)\right]^2, \quad (6.4)$$

where $I_0(x)$, $K_0(x)$ are the modified Bessel functions. The quasimomentum

$$p_{\rm F}^{\rm B} = \tfrac{1}{2}\pi\hbar N_{\rm L}^{\rm B}$$

must satisfy the inequality

$$T \ll (p_{\rm F}^{\rm B})^2/2m < \frac{\hbar^2}{2ma_{\rm B}^2}, \quad (6.5)$$

since we have assumed that only the lowest Landau level is occupied. Here $N_{\rm L}^{\rm B}$ is the electron density per unit length in the magnetic field. The expression (6.4) demonstrates that, provided the magnetic field goes up, the localization radius $a_{\rm B}$ of the wavefunctions suppresses the probability of the backscattering processes. Note that if one assumes $N_{\rm L}^{\rm B} = N_{\rm L}$, where $N_{\rm L}$ is the electron density per unit length for $B = 0$, then the effective interaction depends on the magnetic field only via $a_{\rm B}$. Therefore in this case the magnetic field does not change the magnitude of the transferred momentum, in contrast with the previous case where such a change leads to a rapid increase of the drag current in a strong magnetic field. The drag current is

$$J_{\rm drag} = J_{01}\frac{eV}{T}\left(\frac{T}{\varepsilon_{\rm F}^{\rm B}}\right)^2\left(\frac{U_{12}}{2T}\right)^2\left[\sinh\left(\frac{U_{12}}{2T}\right)\right]^{-2} \tag{6.6}$$

$$J_{01} = -\frac{e^5 m}{2\pi^2\kappa^2\hbar^3}L_z k_{\rm F}^{\rm B} g(2p_{\rm F}^{\rm B}). \tag{6.7}$$

Second, we can compare our results with the drag between two two-dimensional quantum wells [12] in the field-free case. In this case the transresistance ρ_{12} is proportional to

$$\rho_{12} \sim T^2\frac{1}{(k_{\rm S}d)^2}\frac{1}{(k_{\rm F}d)^2}, \tag{6.8}$$

where $k_{\rm S}$ is the single-quantum well (two-dimensional) Thomas–Fermi screening wavevector of the order of the inverse effective Bohr radius, and d is the interwell distance. First we note that the temperature dependence of the Coulomb drag between two (three-dimensional) quantum wells in the strong magnetic fields is weaker than for the drag in two dimensions. Second, we note that in the latter case the contribution from the backscattering processes can be neglected as compared to the small angle scattering contribution with transferred momenta $0 < q < 1/d \ll k_{\rm S}$, while in our case only the backscattering processes are important (this is again a consequence of the one-dimensionality of the Coulomb drag problem in the quantum limit).

To summarize, we have developed a theory of the Coulomb drag between two quantum wells in a strong longitudinal magnetic field. We have considered a comparatively simple limiting case where only the lowest Landau level is occupied. The strong magnetic field makes transverse motion of an electron one dimensional. These one-dimensional electron states can be visualized as quantum 'tubes' or 'wires'. Therefore, the Coulomb drag problem in this situation becomes similar to the Coulomb drag problem between two parallel nanowires.

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References

[1] Bonsager M C, Flensberg K, Hu B Y-K and Jauho A-P 1996 *Phys. Rev. Lett.* **77** 1366
Bonsager M C, Flensberg K, Hu B Y-K and Jauho A-P 1997 *Phys. Rev.* B **56** 10314
[2] Kamenev A and Oreg Y 1995 *Phys. Rev.* B **52** 7516
[3] Hu B Y K 1997 *Phys. Scr.* T **69** 170
[4] Bogachek E N, Jonson M, Shekhter R I and Swahn T 1994 *Phys. Rev.* B **50** 18341
[5] Gurevich V L, Pevzner V B and Fenton E W 1998 *J. Phys.: Condens. Matter* **10** 2551
[6] Gurevich V L and Muradov M I 2000 *Pis. Zh. Eksp. Teor. Fiz.* **71** 164
Gurevich V L and Muradov M I 2000 *JETP Lett.* **71** 111 (Engl. Transl.)
[7] Dubinskaya L S 1969 *Zh. Eksp. Teor. Fiz.* **56** 801
Dubinskaya L S 1969 *JETP* **29** 836 (Engl. Transl.)
[8] Landauer R 1957 *IBM J. Res. Dev.* **1** 233
Landauer R 1989 *IBM J. Res. Dev.* **32** 306
[9] Imry Y 1986 *Directions in Condensed Matter Physics* ed G Grinstein and G Mazenko (Singapore: World Scientific) p 101
[10] Büttiker M 1986 *Phys. Rev. Lett.* **57** 1761
[11] Ivchenko E L, Kiselev A A and Willander M 1997 *Solid State Commun.* **102** 375
[12] Gramila T J, Eisenstein J P, MacDonald A H, Pfeiffer L N and West K W 1991 *Phys. Rev. Lett.* **66** 1216